UDC 539.3 DOI: 10.32326/1814-9146-2022-84-1-15-24

ONE SIMPLE CASE OF LUBRICATED LINE CONTACT FOR DOUBLE-LAYERED ELASTIC SOLIDS*

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Received by the Editor 06.01.2022

The main goal of this paper is to consider formulation and solution of a lubrication problem based on the expressions for elastic surface displacements derived asymptotically from an exact solution for a loaded double coated elastic substrate which are valid within a certain region of the problem input parameters. Therefore, a new relatively simple numerical model of the behavior of lubrication parameters in a line lightly loaded contact of double coated elastic cylinders has been developed. For simplicity materials and coatings of both cylinders are considered identical. The main part of the elastic displacements of the contact surfaces is represented by simple Winkler like contributions. The problem is reduced to a numerical solution of a system of two transcendent equations. The formulas for lubrication parameters such as distributions of contact pressure, gap, lubrication film thickness, shear stress, coefficient of friction, and contact energy loss were derived and used for specific calculations. Generally, compared to lubrication parameters in the contact of rigid solids without coatings the effect of the double coating resulted in reduced (up to 60% or more) contact pressure, increased contact area and film thickness as well as some reduction of frictional forces and energy losses. Some specific results for the obtained solutions are provided.

Keywords: line lightly loaded lubricated contacts, elastic double coatings, asymptotic representations for elastic surface displacements, elastohydrodynamically lubricated contacts.

Introduction

To increase the performance of machine parts and structural elements, protective coatings are usually used. Coatings are able to increase the values of tribological characteristics, reduce friction, protect against corrosion, erosion, temperature effects, etc. There is a need for a theoretical understanding of the contact behavior of coated structural members both in industry and science. In practice, coatings may represent various structures: homogeneous, multi-layered or functionally graded. Dry contact of bodies with homogeneous, functionally graded coatings with and without friction was considered

* This work was supported by Russian Foundation for Basic Research (grant №20-58-53045-GFEN-a) and the National Natural Science Foundation of China (№12011530056).

in [1–5]. Multilayered coatings, in practice, are often created to achieve specified characteristics, for example, to reduce wear [6] or to increase hardness [7]. In such cases, two-layered coatings are of particular interest, since they are easier to create and can form naturally, for example, when oxide films are formed or when lubricant components are absorbed on the coating surface. Thus, the plane contact problem on indentation of a bilayered (two-layered elastic coating coupled with a nondeformable substrate) was studied in frictionless [8] and frictional [9] formulations. The wear of a two-layered coating subjected for heating and friction is considered in [10]. Frictionless indentation and torsion of two-layered coatings on an elastic substrate were studied in [11–13].

Another way to increase performance of machine parts is the use of lubricants in the contacting units. So lubricants are widespread in the automotive industry and are used, in particular, to increase the efficiency of internal combustion engines, because even small decrease in energy losses multiplied by millions of car engines can be quite significant in reducing fuel demand worldwide. Extending product life cycle of engines and reducing emissions due to decreased wear of engine components is another desirable goal of engine designers. Frictional losses in engines are associated with specific engine components including piston ring-liner contacts, journal bearings, tappets etc. The major losses are associated with piston rings (~40%) and skirts. Therefore, understanding tribological characteristics of piston ring-cylinder liner contact and possible ways of reducing the intensity of lubricated contacts may help in reducing frictional losses and increasing fuel economy. The reciprocal motion of piston rings includes several regimes of lubrication set by the linear velocity u and applied load P at a particular piston ring position and viscosity lubricant μ. Near dead zones (where $u \to 0$), frictional losses depend primarily on the nature of boundary lubrication and the formation of so-called tribolayers [14]. In such a situation, the lubrication problem can be reduced to a squeeze flow problem [15]. However, the main viscous energy losses are due to hydrodynamic regime of lubrication in the central portion of piston movement cycle relatively far from the dead zones. In this regime, viscous losses are associated with the specific distribution of pressure within the contact zone, which depends on rheological parameters of lubricants and elastic parameters of the solids in contact. Some understanding of the influence of lubricant additives making lubricant rheology non-Newtonian is presented in [16]. In [17] the elastohydrodynamically lubricated (EHL) model for a coated point contact was introduced. In the paper the numerical results showed that hard coatings increase friction while soft coatings decrease it. A similar problem for solids with multiple coatings (including some functionally graded ones) has been considered numerically in [18]. Kudish et al. [19, 20] considered numerically and asymptotically an EHL problem for line contacts with coated surfaces. The coatings were made of different functionally graded elastic materials. The lubricated contact of a half-plane with a coating under conditions of a non-Newtonian fluid is considered in [21]. Heavily loaded line EHL contacts with thin adsorbed soft layers was considered in [22].

In the present paper, we explore the possible advancements which can be made by using coated solids. To simplify the problem it is considered that the rheology of the lubrication fluid is Newtonian. More specifically, we consider how in some cases double layered elastic coatings may improve such parameters as lubrication film thickness and friction losses. We investigate the case when the elastic moduli of the coatings and substrate are very different in magnitude while the contact pressure created in a lubricated contact is relatively small to appreciably change lubricant viscosity.

1. Main simplified relationships used in the problem formulation

Let us consider a plane problem for a lubricated contact of an infinite cylinder with a half-space (see fig. 1). Both the cylinder and the half-space have attached to them relatively thin elastic double coatings. For simplicity we will assume that the lubricant is a Newtonian incompressible fluid with constant viscosity μ. The coordinate system is introduced in such a way that the x – axis is directed along the lubricant flow and perpendicular to the cylinder axis, the y – axis is directed along the cylinder axis, and the z – axis is directed across the lubricant layer. The cylinder is separated from the surface of the half-space by a continuous lubricant layer. The cylinder steadily rolls and slides in the direction of the x – axis with speed u_2 while the half-space moves in the same direction with speed u_1.

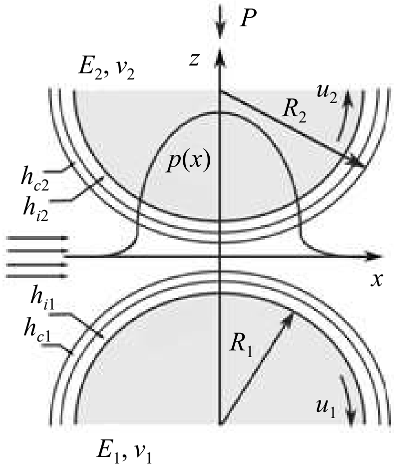

Fig. 1. The general view of a lubricated contact

The components of the lubricant velocity are represented by functions $u(x,y,z)$, $v(x,y,z)$ and $w(x,y,z)$. Due to this problem geometry we have

$$v(x,y,z)=0, \quad \frac{\partial v(x,y,z)}{\partial y}=0.$$

Due to that the problem parameters are independent of the coordinate y. For a typical line concentrated contact the gap between the contact surfaces is much smaller than the contact length. Therefore, the simplified equations of the motion of such a fluid are as follows [23, 24]

$$-\frac{\partial p}{\partial x}+\frac{\partial \tau_{zx}}{\partial z}=0, \quad \frac{\partial p}{\partial z}=0, \quad \tau_{zx}=\mu\frac{\partial u}{\partial z}, \tag{1}$$

where p is the contact pressure.

For an incompressible fluid the continuity equation has the form

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0. \tag{2}$$

The no slip boundary conditions on the fluid speed u and no penetration of the fluid on w at the solid boundaries are as follows

$$u\left(x,-\frac{h(x)}{2}\right)=u_1, \quad u\left(x,\frac{h(x)}{2}\right)=u_2,$$

$$w\left(x,-\frac{h(x)}{2}\right)=-\frac{1}{2}u_1\frac{dh(x)}{dx},\quad w\left(x,\frac{h(x)}{2}\right)=\frac{1}{2}u_2\frac{dh(x)}{dx}, \tag{3}$$

where $h(x)$ is the gap between contact surfaces. The boundary conditions imposed on w are obtained based the fact that in concentrated contacts $dh/dx \ll 1$.

An accurate and precise description of surface normal and tangential displacements for a double layered elastic solid loaded with a normal and tangential surface loads is provided in [25]. We will assume that the substrate material occupying the lower subspace has Young's modulus E_s and Poisson's ratio ν_s while the materials of the upper and intermediate coatings have Young's modulus E_c and E_i and Poisson's ratios ν_c and ν_i, respectively, while their thicknesses are h_c and h_i, respectively. However, the exact expressions for the surface displacements for such elastic solids are very complex. An asymptotic analysis of these expressions in a spectrum of various limiting cases has been conducted in [25] which in some specific limiting cases resulted in a much simpler relationships compared to the original ones. In this paper we will consider just one of such cases of a lightly loaded contact charecterized by a Winkler–Full type relationships for surface displacements U and W called in [25] as Case II, when

$$E_{33}'^{(i)} << E_{33}'^{(c)} << E_{33}'^{(s)}, \tag{4}$$

$$E_{11}'=E_{33}'=\frac{E}{1-\nu^2},\quad E_{13}'=\frac{2E}{(1+\nu)(1-2\nu)}, \tag{5}$$

where superscripts (i), (c), and (s) correspond to the materials of the intermediate and upper coatings as well as the substrate, respectively, E and ν are Young's modulus and Poisson's ratio of the corresponding material.

Specifically, for surface displacements we will consider Case U1U4 for the tangential displacement U

$$\frac{d}{dx}U(x,0)=-B_{13}^{(0)}(2\pi)^{-1}p(x)+\ldots, \tag{6}$$

which is correct if the following relationships are satisfied

$$a_H >> E_{13}'^{(s)}\frac{B_{13}^{(1)}h_c+B_{13}^{(2)}h_i}{4\pi},\quad R >> E_{13}'^{(s)}\frac{B_{11}^{(1)}h_c+B_{11}^{(2)}h_i}{8\pi}, \tag{7}$$

and Case W2W7 for the normal displacement W

$$\frac{d}{dx}W(x,0)=-(2\pi)^{-1}(B_{33}^{(1)}h_c+B_{33}^{(2)}h_i)\frac{dp(x)}{dx}+\ldots \tag{8}$$

is correct if the following relationships are satisfied

$$\frac{a_H}{R}<<2\frac{B_{33}^{(1)}h_c+B_{33}^{(2)}h_i}{B_{13}^{(1)}h_c+B_{13}^{(2)}h_i},\quad a_H<<E_{33}'^{(s)}\left(B_{33}^{(1)}h_c+B_{33}^{(2)}h_i\right), \tag{9}$$

where h_c and h_i are thicknesses of the upper and intermediate coatings, R is the effective radius of a cylinder causing applied to the surfaces normal $-p(x)$ and tangential $\tau_{xz}(x)$ stresses, and a_H is a typical (Hertzian) half-length of a dry contact of elastic solids without coatings, $a_H=2\sqrt{RP/\left(\pi E_{33}'^{(s)}\right)}$, P is the load per unit length applied to the cylinder. Some of the constants involved in the previous formulas are given below [25]

$$B_{13}^{(0)}=\frac{4\pi}{E_{13}'^{(s)}},\quad B_{13}^{(1)}=\frac{8E_{33}'^{(c)}}{E_{13}'^{(c)}E_{33}'^{(s)}},\quad B_{13}^{(2)}=\frac{4(1-2\nu_i)}{(1-\nu_i)E_{33}'^{(s)}},$$

$$B_{33}^{(1)}=\frac{4\pi}{(1-\nu_c)E_{13}'^{(c)}},\quad B_{33}^{(2)}=\frac{4\pi}{(1-\nu_i)E_{13}'^{(i)}}. \tag{10}$$

For simplicity we will assume that the materials of both contact solids are identical and the coating thicknesses of coatings on both solids are also the same, i.e. $h_{c1}=h_{c2}=h_c$ and $h_{i1}=h_{i2}=h_i$. Based on the above formulas the actual surface velocities of the solids are

$$v_i(x)=u_i\left[1+\frac{d}{dx}U_i\left(x,(-1)^i\frac{h(x)}{2}\right)\right],\quad i=1,2. \tag{11}$$

That, finally, allows us to formulate the lubrication problem as follows (see [23, 24])

$$\frac{d}{dx}\left\{\frac{v_1(x)+v_2(x)}{2}h(x)-\frac{h^3(x)}{12\mu}\frac{dp(x)}{dx}\right\}=0,\quad p(x_i)=p(x_e)=0,\quad \frac{dp(x_e)}{dx}=0,$$

$$h=h_e+\frac{x^2-x_e^2}{2R'}+\frac{1}{\pi}\left(B_{33}^{(1)}h_c+B_{33}^{(2)}h_i\right)p(x),\quad \int\limits_{x_i}^{x_e}p(x)dx=P, \tag{12}$$

$$\frac{v_1(x)+v_2(x)}{2}=\frac{u_1+u_2}{2}\left[1-\frac{B_{13}^{(0)}}{2\pi}p(x)\right],$$

where x_i and x_e are the contact inlet and exit point coordinates (x_i is considered to be given while x_e needs to be determined from the problem solution), h_e is lubrication film thickness at the exit point x_e which is also determined from the problem solution, and R' is the effective curvature radius of the contact solids.

By introducing the following dimensionless variables typical for lightly loaded lubricated contacts [23, 24]

$$\{x',a,c\}=\{x,x_i,x_e\}\frac{\theta}{2R'},\quad h'=\frac{h}{h_e},\quad p'=p\frac{\pi R'}{\theta P},$$

$$\mu'=\frac{\mu}{\mu_a},\quad \gamma=\frac{h_e\theta^2}{2R'},\quad \theta^2=\frac{P}{3\pi\mu_a(u_1+u_2)},\quad v_i'=\frac{2v_i}{u_2+u_1},\quad i=1,2, \tag{13}$$

and omitting in the further consider ations primes at the dimensionless variables we obtain the following problem in dimensionless variables

$$\frac{d}{dx}\left\{\left(1-\frac{2}{\pi V\theta}p(x)\right)h(x)-\frac{\gamma^2h^3(x)}{\mu}\frac{dp(x)}{dx}\right\}=0,\quad p(a)=p(c)=\frac{dp(c)}{dx}=0,$$

$$\gamma(h(x)-1)=x^2-c^2+\frac{\sigma}{\pi V}p(x),\quad \int\limits_a^c p(x)dx=\frac{\pi}{2}, \tag{14}$$

$$V=\frac{R'E_{13}'^{(s)}}{\theta^2P},\quad \sigma=\theta\left[\frac{h_c/R'}{(1-\nu_c)E_{13}'^{(c)}/E_{13}'^{(s)}}+\frac{h_i/R'}{(1-\nu_i)E_{13}'^{(i)}/E_{13}'^{(s)}}\right],$$

where V and σ are two given dimensionless parameters. Obviously, the differential (Reynolds) equation can be analytically integrated one time. After that the problem can be reduced to determining constants γ and c from a system of two transcendent equations which needs to be solved numerically using an iterative method analogous to Newton's method.

The lubrication problem for solids with coatings made of different materials and of different thickness can be set up in a similar way. Moreover, the equations for the case when one of the solids does not have coatings coincides with equations (14) in which the dimensionless parameter σ has to be replaced by $\sigma/2$. Therefore, the effect of the coatings is diminished.

On the other hand, the problem described by equations (14) can be solved using the regular perturbation method for $\sigma \sim 1$ and $V >> 1$ presented in [23, 24]. Obviously, for large V as V increases the problem solution approaches the solution of the corresponding lubrication problem for rigid solids [23, 24]. This trend is clear from the numerical data presented below. After the solution is obtain we can calculate the dimensionless friction force $F' = F/P$ (i.e. essentially the friction coefficient). Omitting the prime for the dimensionless F we obtain

$$F = \frac{1}{6\pi\gamma\theta}\int\limits_{a}^{c} \frac{\mu(v_2 - v_1)dx}{h(x)}, \quad v_2 - v_1 = s_0\left[1 - \frac{2}{\pi V\theta}p(x)\right], \quad s_0 = 2\frac{u_2 - u_1}{u_2 + u_1}. \qquad (15)$$

Also, we can calculate the total energy loss E in the lubricated contact as follows

$$E = \int\limits_{x_i}^{x_e} dx \int\limits_{-h(x)/2}^{h(x)/2} \tau_{xz}(x,z)\frac{\partial u(x,z)}{\partial z}dz. \qquad (16)$$

The dimensionless value E' introduced by the relationship $E = E'\mu_a(u_1 + u_2)^2/4$ in dimensionless variables (13) has the form (prime is omitted)

$$E = \frac{\theta\mu}{\gamma}\int\limits_{a}^{c}\left\{\frac{(v_2(x) - v_1(x))^2}{h(x)} + \frac{12\gamma^4 h^3(x)}{\mu^2}\left(\frac{dp(x)}{dx}\right)^2\right\}dx, \qquad (17)$$

where $v_2 - v_1$ is determined in (15).

2. Some results for the lubrication problem

In this section, our goal is to illustrate the developed approach by a specific numerical example. Specifically, we will take $E_c = 112$ GPa, $\nu_c = 0.32$, $E_i = 74$ GPa, $\nu_i = 0.34$, $E_s =$ $= 200$ GPa, $\nu_s = 0.25$, which are typical elastic parameters for titanium, duraluminum, and steel, respectively. Also, it is assumed that the effective radius of contact solids $R' =$ $= 0.01125$ m, the applied force $P = 1.5 \cdot 10^4$ N/m, and the coating thicknesses are taken as follows $h_c = 0.5 \cdot 10^{-7}$ m and $h_i = 0.5 \cdot 10^{-5}$ m. For this set of data all of the conditions (7) and (9) for the validity of the used approximations for the surface displacements U and W are satisfied. The lubrication regime is lightly loaded and, therefore, the lubricant viscosity μ can be considered independent of pressure and equal to the ambient viscosity $\mu_a =$ $= 3.524 \cdot 10^{-3}$ N·s/m^2. The following results are obtained for fixed values of parameters $\mu = 1$, $s_0 = 2$ and varying values of the parameters θ and σ.

Just notice, that for the case of rigid solids without coatings the dimensionless film thickness $\gamma_r = 0.157$ and dimensionless coordinate of the exit point $c_r = 0.170$ (see [23, 24]).

The general trend of the solution of our problem compared to the solution of the similar lubrication problem for rigid solids is as follows. As the dimensionless parameter $\theta^{-2} = 3\pi\mu_a(u_1 + u_2)/P$ increases the dimensionless exit film thickness γ (see fig. 2*a*) and exit coordinate c (see fig. 2*b*) monotonically increase remaining greater than the values of γ_r and c_r for rigid solids without coatings, respectively, while the dimensionless energy loss E (see fig. 3*a*) increases monotonically remaining lower than the energy loss E_r in the contact for rigid solids without coatings.

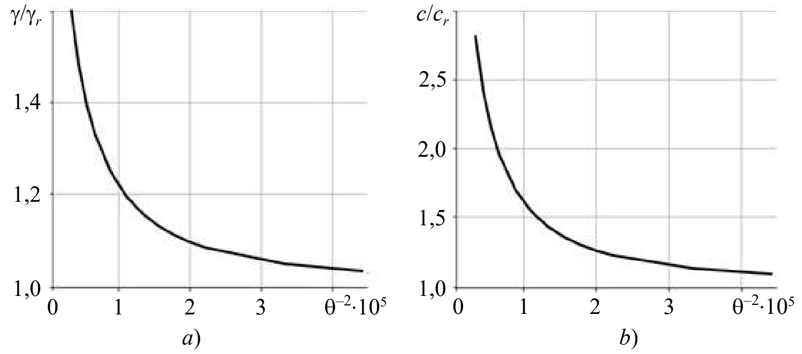

Fig. 2. The graph of the relative exit point:
a) film thickness γ/γ_r versus θ^{-2}; *b*) coordinate c/c_r versus θ^{-2}

The relative friction force F/F_r behaves typical to the well known Stribeck curve (see fig. 3*b*) which for relatively high values of θ^{-2} represents hydrodynamic lubrication regime while for relatively moderate values of θ^{-2} it represents an elastohydrodynamic lubrication regime. As θ^{-2} increases, the behavior of the minimum dimensionless gap $\gamma h_{\min}/(\gamma_r - c_r^2)$ is almost identical to the graph of γ/γ_r versus θ^{-2} (see fig. 2*a*). As θ^{-2} increases the pressure distribution $p(x)$ widens and, in general, gets lower than in the case of the corresponding lubricated contact for rigid solids (see fig. 4).

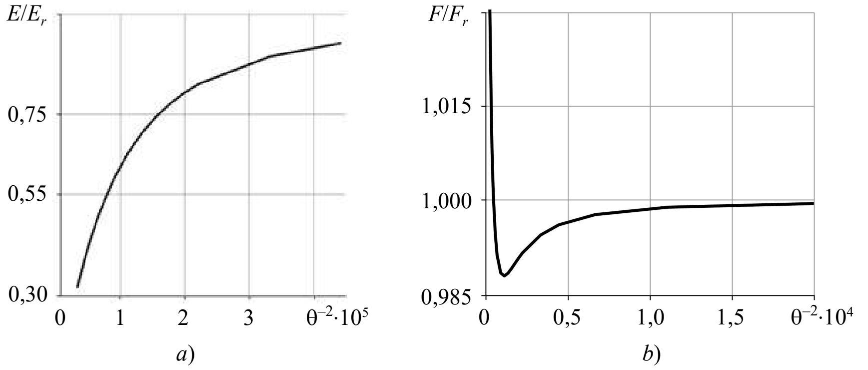

Fig. 3. The graph of the relative:
a) contact energy loss E/E_r versus θ^{-2}; *b*) friction force F/F_r versus θ^{-2}

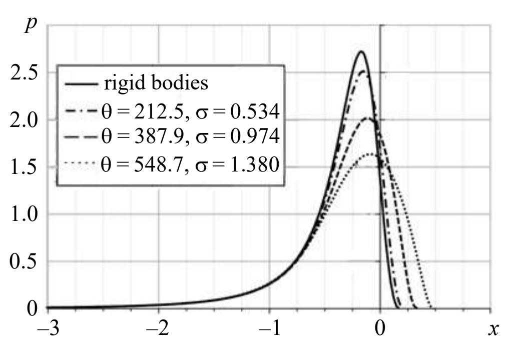

Fig. 4. The graphs of the pressure distributions $p(x)$ versus x for different values of θ, σ

Obviously, the variations of the exit film thickness γ, exit coordinate c, energy loss E, and pressure $p(x)$ with θ compared to their counterparts for just rigid solids can be very significant (up 60% or more) while the variation in the friction force F is within about 2% of the one for the case of lubricated rigid solids (see fig. 2–4).

Closure

A new relatively simple numerical model of the behavior of lubrication parameters in a line contact of double coated elastic cylinder with a half-space has been developed. The main part of the elastic displacements of the contact surfaces is represented by a simple Winkler-Fuss type like contributions. The problem is reduced to a numerical solution of a system of two transcendent equations. The lubrication parameters such as contact pressure, gap, minimum lubrication film thickness, friction force, and energy loss were determined. Generally, the effect of the double coating resulted in significantly reduced (up to 60% or more) contact pressure, increased contact area, and film thickness as well as a reduction of the contact energy loss. The behavior of the friction force resembles the behavior of the Stribeck curve. Some specific results for the obtained solutions are provided.

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ОДИН ПРОСТОЙ СЛУЧАЙ ЛИНЕЙНОГО КОНТАКТА ТЕЛ С ДВУХСЛОЙНЫМИ ПОКРЫТИЯМИ С УЧЕТОМ СМАЗКИ

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Рассмотрены постановка и решение задачи о контакте тел со смазкой на основе выражений для перемещений упругой поверхности, полученных асимптотически из точных представлений для перемещений упругой полуплоскости с двухслойным покрытием. Асимптотические представления перемещений получены для определенной области входных параметров задачи. С использованием этих представлений разработана новая относительно простая численная модель оценки влияния параметров смазки на линейный легко нагруженный контакт упругих цилиндров с двухслойным покрытием. Для простоты материалы и покрытия контактирующих цилиндров считаются идентичными. Основной вклад упругих перемещений поверхности покрытий контактирующих тел представлен простыми выражениями, схожими с соотношениями Винклера. Задача сводится к численному решению дифференциального уравнения относительно неизвестного давления с граничными условиями, характеризующими отсутствие контактного давления на краях области контакта. Уравнение содержит две неизвестные константы: толщину слоя смазки в точке выхода из зоны контакта и координату точки выхода смазки из зоны контакта. Эти константы определяются итерационно из дополнительного интегрального условия и граничного условия на производную от давления в точке выхода смазки из зоны контакта. Получены и применены для численных расчетов формулы для основных параметров, характеризующих контакт тел со смазкой, таких как толщина пленки смазки, силы трения, потери энергии и др. Показано, что для рассмотренного случая, по сравнению со случаем контакта недеформируемых тел, наличие двухслойного покрытия привело к снижению (более чем на 60%) контактного давления, увеличению площади контакта и толщины пленки смазки, а также некоторому снижению силы трения и потери энергии. При численном анализе были рассмотрены конкретные материалы покрытий.

Ключевые слова: легко нагруженный контакт со смазкой, двухслойное упругое покрытие, асимптотические представления для смещений поверхности, упругогидродинамический контакт со смазкой.