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EXPLICIT FORMULA FOR DEPTH OF PENETRATION OF CONE-NOSED IMPACTOR INTO ANISOTROPIC SHIELDS

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The field of application of Functionally Graded Materialsis steadily expanding, which stimulates research in the relevant areas. In relation to penetration mechanics, these are primarily experimental studies of multilayer barriers consisting of plates "in contact" with various mechanical properties. Despite intensive research, explicit formulas for integral penetration characteristics (penetration depth and ballistic limit) cannot be obtained, except for the case when sequential penetration of layers (barriers with large gaps between layers).

In this article, explicit formulas for the depth of penetration into an semi-infinite shield and for the ballistic limit velocity applying penetration into a shield of a finite thickness are derived assuming that the hardness of the barrier material varies continuously depending on barrier depth. The theoretical analysis is based on a model that represents the normal stress at points on the surface of the penetrating body that are in contact with the barrier as a quadratic function of the normal component of local impactor velocity with a zero linear term (the Vitman-Stepanov model). Difference of the dynamic hardness in different points of impactor-barrier contact is taken into account. It is also assumed that the nose of the striker has the form of a straight circular cone and the initial stage of penetration when the striker is not completely immersed in the barrier is ignored.

Keywords: impactor, shield, barrier, penetration, depth, anisotropy.

Introduction

The field of application of Functionally Graded Materialsis steadily expanding, which stimulates research in the relevant areas [1–4]. In relation to penetration mechanics, these are primarily experimental studies of multilayer barriers consisting of plates “in contact” with various mechanical properties, as evidenced by reviews [5–6] and review sections of monographs [7, 8]. The theoretical analysis is based mainly on a model that represents the normal stress at points on the surface of the penetrating body that are in contact with the barrier as a quadratic function of the local normal velocity component with a zero linear term (in Russian-language publications, it is known as the Vitman – Stepanov model [9]). Usually additional simplifications are also accepted, in particular, it is assumed that the nose of the striker has the form of a straight circular cone, the specifics of the initial and final stages of penetration are ignored, when the striker is not completely immersed in the barrier, and others. Despite intensive research [10–20], explicit formulas for integral penetration characteristics (penetration depth and ballistic limit) cannot be obtained, except for the case when sequential penetration of layers (barriers with large gaps between layers) is actually assumed [7, 10].

This article is close to the work [20] on the approach to the problem. However, unlike [20], impactors that have a conical shape of the nose are considered, another approximation of the dependence of hardness on depth is used and difference of the dynamic hardness in different points of impactor-barrier contact is taken into account. This results in an explicit formula for the penetration depth with continuously varying hardness of the shield. The description of the interaction of the barrier with the striker is based on the Vitman – Stepanov model; the effects associated with incomplete immersion of the striker in the barrier at the initial stage are not taken into account .

1. Mathematical model and statement of the problem

Consider high speed normal penetration of a rigid, sharp (pointed) body of revolution into a shield with hardness, depending on the depth of the semi-infinite shield. It is assumed that the interaction between the projectile and the shield is described by the following two-term model what is known as the Vitman – Stepanov model [9]:

$$d\mathbf{F} = [\gamma v_n^2 + Y]\mathbf{n}^0 dS, \tag{1}$$

where γ is density of the shield's material; Y is a parameter that determines mechanical properties of the material (dynamic hardness); $d\mathbf{F}$ is the force acting on the lateral surface element dS of a projectile that is in contact with the plate (barrier), $\mathbf{n}^0$ and v_n are the inner normal unit vector and normal component of the instantaneous velocity of the projectile $\mathbf{v}$ at a given location on the projectile surface, correspondingly.

The resultant force acting on the projectile at each instant of time is determined by integrating $d\mathbf{F}$ over the lateral surface of the projectile-barrier contact at the same instant, S_{lat}. Then formula for the drag force, D is the following:

$$D = (-\mathbf{v}^0)\iint_{S_{\text{lat}}} d\mathbf{F} = \iint_{S_{\text{lat}}} (-\mathbf{v}^0) d\mathbf{F}, \tag{2}$$

where $\mathbf{v}^0$ is unit vector of the velocity of the impactor.

Hereafter we use the following notations. The coordinate h, the instantaneous depth of penetration, is defined as the distance between the nose of the impactor and the front surface of the barrier. The cylindrical coordinates x, ρ, ϕ with the origin in the nose of the impactor are associated with it whereas the coordinate x directs along the axis of the impactor. The equation $\rho = \Phi(x)$, where Φ is a convex function, determines the shape of the impactor's surface, L is the length of the nose of the impactor which interacts with the barrier; the impactor has also the cylindrical part of the length L_0. Note that the section $x = x_0$ is located at the depth $h_0 - x_0$ if the nose of the impactor is situated at the depth $h = h_0$.

Neglecting the effect associated with the partial immersion of the impactor in the barrier at the initial stage of penetration, one can obtain the following relationship for D in the case of the model given by Eq. (1) [7, 8]:

$$D(h, v) = f_2 v^2 + f_0(h), \tag{3}$$

where

$$f_0(h) = 2\pi \int_0^L Y(h-x)\Phi_x \Phi dx, \quad f_2 = 2\pi\gamma \int_0^L \frac{\Phi_x^3 \Phi}{\Phi_x^2 + 1} dx, \quad \Phi_x = \frac{d\Phi}{dx}. \tag{4}$$

Then the equation of motion of impactor with the mass m is the following:

$$\frac{m}{2}\frac{dv^2}{dh} + f_2 v^2 + f_0(h) = 0. \tag{5}$$

Solution of this linear relative v^2 differential equation with condition $v(0) = v_{\text{imp}}$, where v_{imp} is impact velocity, can be written in the form:

$$v^2(h) = \frac{1}{Q(h)}\left[v_{\text{imp}}^2 - \frac{2}{m}\int_0^h f_0(\tilde{h}) Q(\tilde{h})\, d\tilde{h} \right], \tag{6}$$

where

$$Q(h) = \exp\left(\frac{2}{m}\int_0^h f_2 d\zeta \right) = \exp\left(\frac{2 f_2 h}{m} \right). \tag{7}$$

Using the condition $v(H) = 0$, we obtain the equation for determining the dept of penetration (DOP) H:

$$\int_0^H f_0(h) Q(h)\, dh = \frac{m}{2} v_{\text{imp}}^2. \tag{8}$$

Our goal is to obtain an explicit formula for the DOP.

2. Solution of the problem

For more clarity, we consider conical-nosed impactors. In this case the equation of impactor's generatrix is written as follows:

$$\Phi(x) = \operatorname{tg}\vartheta x, \tag{9}$$

where ϑ is half apex angle of the cone and

$$f_0(h) = 2\pi \operatorname{tg}^2\vartheta \int_0^L Y(h-x) x dx, \quad f_2 = \pi\gamma \operatorname{tg}^4\vartheta \cos^2\vartheta L^2. \tag{10}$$

Then Eq. (8) is transformed to the form:

$$\int_0^H dh \int_0^L dx [x Y(h-x)\exp(k_2 h)] = k_1 v_{\text{imp}}^2, \tag{11}$$

where

$$k_1 = \frac{m}{2\pi \operatorname{tg}^2\vartheta}, \quad k_2 = \frac{2\pi\gamma \operatorname{tg}^4\vartheta \cos^2\vartheta L^2}{m}. \tag{12}$$

Taking into account that

$$m = \pi\gamma \operatorname{tg}^2\vartheta L^3\left(\frac{1}{3} + \overline{L}_0 \right), \quad \overline{L}_0 = \frac{L_0}{L}, \tag{13}$$

the expression for k_2 can be rewritten as follows:

$$k_2 = \frac{2\sin^2\vartheta}{L(1/3+\bar{L}_0)}. \tag{14}$$

For function $Y(z)$ that describes dynamic hardness Y depending on the depth of the barrier z one can select the approximation [2] with empirical coefficients α and β:

$$Y(z) = \alpha\exp(\beta z) \tag{15}$$

that represent quit well the possible types of behavior of dependence $Y(z)$. Then Eq. (11) takes the form:

$$\int_0^H \exp(k_3 h)dh \int_0^L x\exp(-\beta x)dx = \frac{k_1}{\alpha}v_{\text{imp}}^2. \tag{16}$$

Calculating integrals in Eq. (16) we obtain the equation:

$$\frac{1}{k_3}[\exp(k_3 H)-1]\frac{1-(L\beta+1)\exp(-\beta L)}{\beta^2} = \frac{k_1}{\alpha}v_{\text{imp}}^2, \tag{17}$$

which implied the solution:

$$H = \frac{1}{k_2+\beta}\ln\left(1+k_3 v_{\text{imp}}^2\right), \tag{18}$$

where

$$k_3 = \frac{k_4}{k_5}, \quad k_4 = (k_2+\beta)\beta^2\frac{k_1}{\alpha}, \quad k_5 = 1-(\beta L+1)\exp(-\beta L). \tag{19}$$

In the case of homogeneous media, $Y(z) = Y(0) =$ const, $\alpha = Y(0) = Y$, $\beta = 0$. Since uncertainty 0/0 appears in the right side of Eq. (19) when $\beta = 0$, we use the L'Hospital rule. Then

$$\frac{dk_4/d\beta}{dk_5/d\beta} = \frac{\beta(2k_2+3\beta)(k_1/Y)}{\beta L^2\exp(-\beta L)} \to \frac{2k_1k_2}{YL^2} \tag{20}$$

and

$$\frac{H}{L} = \frac{1}{k_2 L}\ln\left(1+\frac{2k_1k_2}{YL^2}v_{\text{imp}}^2\right) = \frac{1/3+\bar{L}_0}{2\sin^2\vartheta}\ln\left(1+\frac{2\gamma\sin^2\vartheta}{Y}v_{\text{imp}}^2\right). \tag{21}$$

Consider penetration into a semiinfinite barrier of thickness b. The Army ballistic limit v_{bl} (BLV) is defined as the minimum impact velocity at which the striker reaches the back side of the barrier [21]. Then replacing in equation (18) H by b and v_{imp} by v_{bl} we obtain the explicit formula for the BLV:

$$v_{\text{bl}}^2 = \frac{1}{k_3}\left[\exp(b(k_2+\beta))-1\right]. \tag{22}$$

3. Concluding remarks

Further research within the framework of the developed approach is intended in the following areas: consideration of strikers with a nose in the form of pramids and non-conical bodies of revolution (including impactord with flat blunting); taking into account in the model the specificity of the initial stage of penetration with incomplete immersion of the striker in the barrier and friction between the striker and the barrier; use of other models describing the dependence of hardness on the depth of the barrier.

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ЯВНАЯ ФОРМУЛА ДЛЯ ГЛУБИНЫ ПРОНИКАНИЯ УДАРНИКА С КОНИЧЕСКОЙ ГОЛОВНОЙ ЧАСТЬЮ В АНИЗОТРОПНУЮ ПРЕГРАДУ

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Область применения анизотропных материалов неуклонно расширяется, что стимулирует исследования в соответствующих областях. Применительно к механике проникновения это прежде всего экспериментальные исследования многослойных барьеров, состоящих из пластин с различными механическими свойствами, контактирующих друг с другом. Несмотря на интенсивные исследования, явные формулы для интегральных характеристик проникания (глубина проникания и баллистический предел) получить не удается за исключением случая, когда фактически предполагается, что слои пробиваются последовательно (барьеры с большими зазорами между слоями).

В статье получены явные формулы для глубины проникания в полубесконечную преграду и для баллистической предельной скорости проникания в преграду конечной толщины в предположении, что твердость материала барьера непрерывно изменяется в зависимости от глубины преграды. Теоретический анализ основан на модели, представляющей нормальное напряжение в точках поверхности проникающего тела, контактирующих с барьером, как квадратичную функцию нормальной составляющей локальной скорости ударника с нулевым линейным членом (модель Витмана – Степанова). Учитывается разница динамической твердости в различных точках ударного или барьерного контакта. Также предполагается, что нос бойка имеет форму прямого круглого конуса и игнорируется начальная стадия проникания, когда ударник не полностью погружен в преграду.

Ключевые слова: ударник, преграда, барьер, проникание, глубина, анизотропия.