UDC 624.155 DOI: 10.32326/1814-9146-2019-81-2-233-239

A MODEL OF PILE DRIVING BY MULTIPLE IMPACTS OF FALLING HAMMER

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Received by the Editor 2019/06/07

Calculations related to pile driving play an important role in construction. Therefore, corresponding processes are controlled by regulatory documents, including the necessary formulas, which are accompanied by numerous tables and graphs, allowing the use of techniques in specific conditions of construction. The basis of such applied methods are the results of research in mechanics; the results of such studies are widely presented in the literature. This article is made within the framework of such studies.

The article presents an approximate approach based on the evaluation of the real values of the parameters that determine the process of driving the pile. The proposed approach allowed to obtain an approximate formula to calculate the motion of the pile in soil by successive blows of the hammer falling freely. The following ways to analyze and improve models within this approach are outlined: (1) analysis of the accuracy of the proposed model by its presentation with the results of experiments and calculations using other models as well as with recommendations of normative instruments; (2) generalization of the model for the case when the mechanical properties of the soil vary depending on the depth of immersion into the soil.

Keywords: ground, pile, penetration, hammer, driving, modeling, calculation.

Introduction

Calculations related to pile driving play an important role in construction. Therefore, corresponding processes are controlled by regulatory documents, including the necessary formulas, which are accompanied by numerous tables and graphs [1, 2], allowing the use of techniques in specific conditions of construction. The basis of such applied methods are the results of research in mechanics; the results of such studies are widely presented in the literature (for instance, [3–13]). This article is made within the framework of such studies.

1. Formulation of the problem and mathematical model

Consider a pile with a square cross section and pyramidal shape of the nose driving into soil by falling hammer (Fig. 1). When the nose is submerged in the ground to a depth of h and the hammer falls on the head of the pile from a height H the energy conservation equation can be written as follows:

$$MgH = Mg[L - h^{(i)} + e] + 0.5MV^{(i)2}, \tag{1}$$

where M is mass of hammer, H is height of center of mass of hammer at beginning of fall, g is acceleration of gravity, L is pile length including nose, $h^{(i)}$ is distance from nose of pile to surface of soil prior to its movement after i^{th} impact, e is half the height of hammer, $V^{(i)}$ is speed of hammer before i^{th} blow on pile.

Assuming that after the inelastic impact hammer and the pile continue to move as a single unit one can write:

$$MV^{(i)} = (M + m)v^{(i)}, \tag{2}$$

where m is mass of pile, $v^{(i)}$ is initial speed of hammer-pile construction after i^{th} impact on pile.

Eliminating $V^{(i)}$ from Eqs. (1) and (2), we obtain:

$$w^{(i)} = \gamma_1 h^{(i)} + \gamma_0, \tag{3}$$

where $w^{(i)} = v^{(i)2}$ and

$$\gamma_1 = 2g\left(\frac{M}{M+m}\right)^2, \quad \gamma_0 = 2g\left(\frac{M}{M+m}\right)^2 [H - L - e]. \tag{4}$$

Similar to [10], we use the local interaction model [14] in the version of Vitman and Stepanov [15] to calculate the resistance of the nose of the pile:

$$p = Y_{soil} + \rho_{soil} v_n^2, \tag{5}$$

where p is pressure on nose surface of pile, Y_{soil} is dynamic hardness of soil, ρ_{soil} is density of soil, v_n is normal component of speed on nose of pile.

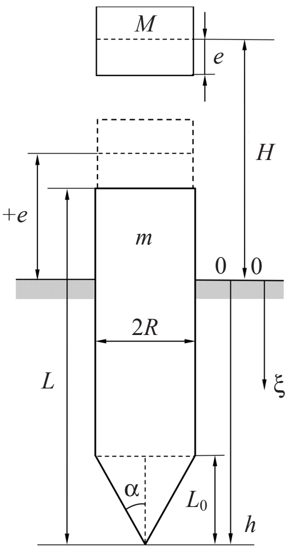

Fig. 1. Notations

After simple algebra we obtain formula for drag force of the pile nose:

$$D_{nose}(w) = k_1 R^2 (Y_{soil} + \rho_{soil} \sin^2 \alpha w), \quad k_1 = 4, \tag{6}$$

where D_{nose} is nose contribution to resistance of pile to penetration, k_1 is parameter that determines shape of nose of pile ($k_1 = 4$), R is half side of square in cross section of pile, ρ_{pile} is average density of pile material, α is angle between side face and axis of the pyramid in nose of pile.

Resistance of the lateral surface of the pile is generally calculated as follows:

$$D_{lat}(h) = k_2 R \int_0^{h-L_0} p_{lat}(\xi)\mu_{lat}(\xi)d\xi, \quad k_2 = 8, \tag{7}$$

where D_{lat} is contribution to resistance of pile to penetration from side surface, h is distance from head of pile to ground surface, k_2 is perimeter of square in cross section of pile ($k_2 = 8$), L_0 is length of nose of pile ($L_0 = R\text{ctg}\,\alpha$), p_{lat} is ground pressure on the side surface of the pile, ξ is coordinate measured from ground surface along direction of penetration, μ_{lat} is coefficient of friction in interaction of side surface of pile with ground.

The equation of pile motion can be written in the following way:

$$0.5(M+m)\frac{dw}{dh} = (M+m)g - D_{nose}(w) - D_{lat}(h), \quad w = v^2 \tag{8}$$

or after substitution D_{nose} from Eq. (6) and simple transformations in the form:

$$\frac{dw}{dh} + fw = G(h), \tag{9}$$

where v is instantaneous velocity of pile and

$$f = \frac{2k_1 R^2 \rho_{soil} \sin^2\alpha}{M+m}, \tag{10}$$

$$G(h) = -\left[-2g + \frac{2k_1 R^2 Y_{soil}}{M+m} + \frac{2}{M+m}D_{lat}(h)\right], \tag{11}$$

Our immediate task is to obtain a relationship between $h^{(i)}$ and $h^{(i+1)}$ ($i = 1, 2, \ldots$). This relation can be obtained by writing the solution of Eq. (9) at two points:

$$w(h^{(i+1)}) = 0, \quad w(h^{(i)}) = \gamma_1 h^{(i)} + \gamma_0. \tag{12}$$

Eq. (9) is a linear equation of the first order and its general solution is known; in relation to the problem of modeling pile driving it can be found, for example, in [10, 11].

2. Investigation of the problem

The solution of Eq. (9) with initial condition $w(h^{(i+1)}) = 0$ can be written as follows:

$$w(h) = \mathrm{e}^{-F(h)} \int_{h^{(i+1)}}^{h} G(\tilde{h})\mathrm{e}^{F(\tilde{h})}d\tilde{h}, \quad F(h) = \int_{h^{(i+1)}}^{h} f dx = f[h - h^{(i+1)}], \tag{13}$$

or

$$w(h) = \mathrm{e}^{-f[h-h^{(i+1)}]}I(h), \quad I(h) = \int_{h^{(i+1)}}^{h} G(\tilde{h})\mathrm{e}^{f[\tilde{h}-h^{(i+1)}]}d\tilde{h}. \tag{14}$$

Then the second relation in Eq. (12) that is written in the form

$$\gamma_1 h^{(i)} + \gamma_0 = \mathrm{e}^{-f[h^{(i)} - h^{(i+1)}]} I(h^{(i)}) \tag{15}$$

sets the desired relationship between $h^{(i)}$ and $h^{(i+1)}$.

To obtain this dependence in explicit form, we consider below the case when p_{lat} and μ_{lat} do not change in the process of pile penetration, i.e.,

$$p_{lat}(\xi) = \text{const}, \quad \mu_{lat}(\xi) = \text{const}. \tag{16}$$

Then

$$D_{lat}(h) = k_2 R p_{lat} \mu_{lat} (h - L_0) \tag{17}$$

and $G(h)$ can be represented in the form:

$$G(h) = \omega_0 + \omega_1 h, \tag{18}$$

where

$$\omega_0 = \left[2g - \frac{2k_1 R^2 Y_{soil}}{M+m} + \frac{2k_2 R p_{lat} \mu_{lat} L_0}{M+m} \right], \quad \omega_1 = -\frac{2k_2 R p_{lat} \mu_{lat}}{M+m} < 0. \tag{19}$$

Integral $I(h)$ in Eq. (14) can be easily calculated; as a result, we obtain:

$$I(h^{(i)}) = \mathrm{e}^{f\cdot[h^{(i)} - h^{(i+1)}]} \left(\frac{\omega_0}{f} + \frac{\omega_1 h^{(i)}}{f} - \frac{\omega_1}{f^2} \right) - \left(\frac{\omega_0}{f} + \frac{\omega_1 h^{(i+1)}}{f} - \frac{\omega_1}{f^2} \right). \tag{20}$$

Since

$$V_{pile} = k_3 (L - L_0) + \frac{1}{3} k_3 L_0 = k_3 L - \frac{2}{3} k_3 L_0 = k_3 R^2 L \left(1 - \frac{2}{3} \cdot \frac{L_0}{L} \right) \approx k_3 R^2 L, \quad k_3 = 4, \tag{21}$$

where k_3 is cross sectional area of pile, V_{pile} is volume of pile,

$$m = \rho_{pile} V_{pile}, \tag{22}$$

and in most cases

$$\frac{\Delta h^{(i)}}{L} < 0.1, \quad \frac{\rho_{soil}}{\rho_{pile}} < 1, \quad \frac{1}{M/m + 1} < 0.5, \quad 2\sin^2 \alpha < 1, \tag{23}$$

the following estimation is valid for $f\,\Delta h^{(i)}$ $\left(\Delta h^{(i)} = h^{(i+1)} - h^{(i)}\right)$:

$$f\,\Delta h^{(i)} = \frac{\Delta h^{(i)}}{L} \cdot \frac{\rho_{soil}}{\rho_{pile}} \cdot \frac{m}{M+m} \cdot \frac{2k_1 R^2 L \sin^2 \alpha}{V_{pile}} \approx$$

$$\approx \frac{\Delta h^{(i)}}{L} \cdot \frac{\rho_{soil}}{\rho_{pile}} \cdot \frac{1}{M/m+1} 2\sin^2 \alpha << 1. \tag{24}$$

Thus

$$\mathrm{e}^{-f\,\Delta h^{(i)}} \approx 1 - f\,\Delta h^{(i)}. \tag{25}$$

Substituting expression for $\mathrm{e}^{-f\,\Delta h^{(i)}}$ from Eq. (25) to Eq. (20) we obtain after some algebra:

$$I(h^{(i)}) \approx (1 - f\Delta h^{(i)})\left(\frac{\omega_0}{f} + \frac{\omega_1 h^{(i)}}{f} - \frac{\omega_1}{f^2}\right) - \left(\frac{\omega_0}{f} + \frac{\omega_1 h^{(i+1)}}{f} - \frac{\omega_1}{f^2}\right) =$$

$$= -\Delta h^{(i)}(\omega_0 + \omega_1 h^{(i)}). \tag{26}$$

Eqs. (15), (25) and (26) imply:

$$(1 - \Delta h^{(i)})(\gamma_1 h^{(i)} + \gamma_0) = -\Delta h^{(i)}(\omega_1 + \omega_2 h^{(i)}). \tag{27}$$

Eq. (27) allows us to write a desired relationship:

$$\Delta h^{(i)} = \frac{\gamma_1 h^{(i)} + \gamma_0}{\gamma_1 h^{(i)} + \gamma_0 - (\omega_0 + \omega_1 h^{(i)})}. \tag{28}$$

The breakout process stops when $\Delta h^{(i)} < \varepsilon_1$ or $|\Delta h^{(i)} - \Delta h^{(i-1)}| < \varepsilon_2$ for some *i*; $\varepsilon_1, \varepsilon_2$ – pre-set parameters, criteria for stopping the process of pile driving.

3. Concluding remarks

This article presents an approximate approach based on the evaluation of the real values of the parameters that determine the process of driving the pile. Future research under this approach should be developed in the following priority areas:

1) analysis of the accuracy of the proposed model by its presentation with the results of experiments and calculations using other models as well as with recommendations of normative instruments;

2) generalization of the model for the case when the mechanical properties of the soil vary depending on the depth of immersion into the soil.

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МОДЕЛЬ ЗАБИВАНИЯ СВАЙ МНОГОКРАТНЫМИ УДАРАМИ ПАДАЮЩЕГО МОЛОТА

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Расчеты, связанные с забивкой свай, играют важную роль в строительстве. Поэтому соответствующие процессы контролируются нормативными документами, в том числе необходимыми формулами, которые сопровождаются многочисленными таблицами и графиками, позволяющими использовать методики в конкретных условиях строительства. Основой таких прикладных методов являются результаты исследований в области механики. Результаты этих исследований широко представлены в литературе. Данная статья выполнена в рамках таких исследований.

В статье представлен приближенный подход, основанный на оценке реальных значений параметров, определяющих процесс забивки сваи. Предложенный подход позволил получить приближенную формулу для расчета движения сваи в грунте вследствие последовательных ударов свободно падающего молота. Намечены пути совершенствования моделей в рамках этого подхода. Исследования рекомендуется развивать в следующих приоритетных областях: анализ точности предлагаемой модели путем ее сопоставления с результатами экспериментов и расчетов с использованием других моделей, а также с рекомендациями нормативных документов; обобщение модели на случай, когда механические свойства почвы изменяются в зависимости от глубины погружения в грунт.

Ключевые слова: грунт, свая, проникание, молот, забитие, моделирование, расчет.