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GENERATION OF PERTURBATIONS BY A FOCUSED SOURCE DRIVING WITH CONSTANT DOWN SPEED ALONG THE BORDER OF THE GRADIENT-ELASTIC HALF-SPACE*

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Within the mathematical model of the gradient-elastic continuum, i.e. medium, the stress-strain state of which is described by the strain tensor, the second gradients of the displacement vector, the asymmetric stress tensor and the couple-stress tensor, the problem of generating perturbations by a moving source is considered. The model under study belongs to the class of generalized continua, its appearance is associated with the names of J.-M. Le Roux and T. Jaramillo. The well-known model of the Cosserat continuum reduces to the model of a gradient-elastic medium when the dependence of the rotation vector on the displacement rotor (constrained rotation) is rigidly fixed in it. It is assumed that the source moves at a constant velocity along the boundary of the half-space. The problem is considered in a two-dimensional formulation, when all the processes are homogeneous along the horizontal transverse coordinate axis. The displacement vector contains two components: longitudinal and vertical transverse. The velocity of the source does not exceed in its magnitude the velocity of the shear elastic wave (subsonic motion). As a result of analytical studies it is shown that a moving source will generate waves propagation along the boundary of a half-space and exponentially decreasing in its depth. The transverse component of the displacement vector always exceeds the longitudinal component, and the rotation of the particles during the propagation of the perturbation occurs along an elliptical trajectory. Such a wave, unlike the classical Rayleigh surface wave, has dispersion, since its phase velocity is not a constant, but depends on the frequency. The amplitudes of displacements vary depending on the magnitude of the load of the moving source and its speed. As the velocity of the source approaches the velocity of the shear wave, the perturbation amplitudes increase without limit.

Keywords: gradient-elastic half-space, moving source, surface wave.

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Introduction

It was shown in [1] that a concentrated source, moving at a constant speed along the boundary of an elastic half-space, can generate surface Rayleigh waves.

Along with the classical continuum models, the generalized continua models are widely used in the mechanics of deformable solid body [2–12].

The gradient-elastic medium, in particular, belongs to the generalized continua model. The appearance of this model dates back to the beginning of the 20th century and is associated with the names of J.-M. Loru [13, 14] and T. Jeremillo [15].

The famous Cosserat continuum model [16] also reduces to the gradient-elastic medium model, when the dependence of the rotation vector on the displacement rotor (constrained rotation) is rigidly fixed [17, 18].

The main laws governing the propagation of a surface wave along the boundary of a gradient-elastic half-space were studied in [19]. It was shown, that such a wave, unlike the classical Rayleigh wave, has a dispersion.

Surface waves propagating along the boundaries of other generalized continua behave in a similar way: Cosserat medium [20–24], porous-elastic medium [25–28]. In the present paper, the problem of a surface wave generated by a source moving along the boundary of a gradient-elastic half-space is considered, and the dependence of the amplitude of a wave on the magnitude of the load of the source and its speed is determined.

1. The basic equations of the gradient elasticity theory

The deformed state of a gradient-elastic medium is described by the strain tensor ε_{kl} and the second gradients of the displacement vector χ_{klm}:

$$\varepsilon_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right),$$
$$\chi_{klm} = -\frac{\partial^2 u_k}{\partial x_l \partial x_m}. \tag{1}$$

When considering the adiabatic processes of elastic deformation, it is necessary to postulate the dependence of the internal energy U on the invariants of the strain measures (1).

We expand the function U in the neighborhood of the natural state ($\varepsilon_{kl} = 0$, $\chi_{klm} = 0$) in a Taylor series, the values of the third order being neglected. For an isotropic homogeneous and centrally-symmetric body we obtain the decomposition of the following form [18]:

$$U = \frac{\lambda}{2}\varepsilon_{kk}^2 + \mu\varepsilon_{ik}^2 + 2\mu l^2(\chi_{klm}^2 + \tilde{\nu}\chi_{klm}\chi_{lkm}), \tag{2}$$

where λ and μ are elastic Lame constants, l^2 is the ratio of the curvature modulus to the shear modulus μ having the dimension of the length squared, $\tilde{\nu}$ is a dimensionless constant, indexes $i, k, l, m = \overline{1, 3}$.

In displacements, the vector equation of the dynamics of a gradient-elastic medium has the form:

$$\rho\ddot{\mathbf{u}} - (\lambda + \mu)\text{grad div}\mathbf{u} - \mu\Delta\mathbf{u} + 4\mu l^2\Delta(\Delta\mathbf{u} + \tilde{\nu}\text{grad div}\mathbf{u}) = 0, \tag{3}$$

where ρ is the density of the medium.

This equation contains the fourth order of derivatives with respect to coordinates, in contrast to the classical Lame equation, which describes the dynamics of a deformable solid body containing second derivatives with respect to coordinates.

2. Problem statement

We consider the propagation of disturbances in the elastic isotropic half-space $y' \geq 0$, caused by a source, moving in the direction of the x' axis with a constant speed D and creating normal load P.

The dynamic equations equivalent to the vector equation (3) in the two-dimensional case (that is, when all processes are homogeneous along z' axis) has the form:

$$\begin{gathered}\frac{\partial \sigma_{x'x'}}{\partial x'} + \frac{\partial \sigma_{y'x'}}{\partial y'} = \rho \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial \sigma_{x'y'}}{\partial x'} + \frac{\partial \sigma_{y'y'}}{\partial y'} = \rho \frac{\partial^2 v}{\partial t^2}.\end{gathered} \tag{4}$$

Where u, v are longitudinal and transverse components of the displacement vector.

The components of the stress tensor are expressed in terms of u, v (where $\sigma_{x'y'} \neq \sigma_{y'x'}$) by the following relations:

$$\begin{gathered}\sigma_{x'x'} = \lambda\left(\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'}\right) + 2\mu \frac{\partial u}{\partial x'}, \quad \sigma_{y'y'} = \lambda\left(\frac{\partial u}{\partial x'} + \frac{\partial v}{\partial y'}\right) + 2\mu \frac{\partial v}{\partial y'}, \\ \sigma_{x'y'} = \mu\left[\frac{\partial v}{\partial x'} + \frac{\partial u}{\partial y'} - l^2\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}\right)\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right)\right], \\ \sigma_{y'x'} = \mu\left[\frac{\partial v}{\partial x'} + \frac{\partial u}{\partial y'} + l^2\left(\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2}\right)\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right)\right].\end{gathered} \tag{5}$$

Couple stresses $\mu_{x'}$ and $\mu_{y'}$ may be also expressed in terms of u, v:

$$\mu_{x'} = 2\mu l^2 \frac{\partial}{\partial x'}\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right), \quad \mu_{y'} = 2\mu l^2 \frac{\partial}{\partial y'}\left(\frac{\partial v}{\partial x'} - \frac{\partial u}{\partial y'}\right). \tag{6}$$

Boundary conditions (4) will have the following form:

$$\sigma_{y'y'} = -P\delta(x' - Dt), \quad \sigma_{y'x'} = 0, \quad \mu_{y'} = 0. \tag{7}$$

3. Problem solution

Let us consider the case when a source of perturbations travels at the speed D less than the speed of the longitudinal wave $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ and shear wave $c_2 = \sqrt{\mu/\rho}$ (a subsonic case).

We introduce a moving coordinate system (x, y) in which the source of perturbations rests and which is connected with a fixed coordinate system by the well-known Galilean transformation:

$$x = x' - Dt, \quad y = y'. \tag{8}$$

Relations (5), (6), (8) allow to write the dynamical equation (4) in displacements:

$$(\lambda+2\mu-\rho D^2)\frac{\partial^2 u}{\partial x^2}+\lambda\frac{\partial^2 v}{\partial x\partial y}+\mu\left[\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+l^2\Delta\left(\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}\right)\right]=0,$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\lambda\frac{\partial^2 u}{\partial x\partial y}+\mu\left[\left(1-\frac{\rho D^2}{\mu}\right)\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}-l^2\Delta\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}\right)\right]=0. \quad (9)$$

Solution to equations (9) we will find in the form:

$$u=A\mathrm{e}^{kqy}\sin kx,\quad v=B\mathrm{e}^{kqy}\cos kx. \quad (10)$$

Inserting (10) in (9) leads to a system of algebraic equations relatively A and B:

$$[-\mu l^2k^2q^4+\mu q^2(1+l^2k^2)-(\lambda+2\mu-\rho D^2)]+[\mu l^2k^2q^3+q(\lambda+\mu-\mu l^2k^2)]B=0,$$

$$[-\mu l^2k^2q^3-q(\lambda+\mu-\mu l^2k^2)]A+[(\lambda+2\mu-\rho D^2)q^2-(\mu+\mu l^2k^2-\rho D^2)]B=0, \quad (11)$$

having non-zero solutions when equalizing:

$$(l^2k^2-\gamma-\beta_1^2)l^2k^2q^6+(l^2k^2(1+l^2k^2-\beta_1^2)+\gamma(1+l^2k^2)(1-\gamma\beta_1^2)+$$
$$+2l^2k^2(\gamma-l^2k^2))q^4+((1+l^2k^2)(1+l^2k^2-\beta_1^2)-(\gamma-\beta_1^2)^2+$$
$$+(\gamma-l^2k^2)^2)q^2+(\gamma-\beta_1^2)(1+l^2k^2-\beta_1^2)=0. \quad (12)$$

Here $\beta_1^2=\rho D^2/\mu$ is the ratio of the source velocity squared to velocity squared of a shear wave; $\mu=(\lambda+2\mu)/\mu$ is the ratio of the velocity squared of a longi-tudinal wave to the velocity squared of a shear wave.

Roots of equation (12) q_1, q_2, q_3 should be sought at Re $q>0$. Longitudinal and transverse displacements will be expressed as sums:

$$u=\sum_{i=1}^{3}\int_0^\infty A_i\mathrm{e}^{kq_iy}\sin kx\,dk,\quad v=\sum_{i=1}^{3}\int_0^\infty \alpha_iA_i\mathrm{e}^{kq_iy}\cos kx\,dk. \quad (13)$$

Here

$$\alpha_i=\frac{\mu l^2k^2q_i^4-\mu q_i^2(1+l^2k^2)+(\lambda+2\mu-\rho D^2)}{\mu l^2k^2q_i^3+(\lambda+\mu-\mu l^2k^2)q_i}. \quad (14)$$

Inserting (13) into boundary conditions (7), we obtain the system of three equations to define A_1, A_2, A_3:

$$\sum_{i=1}^{3}[\lambda+\alpha_iq_i(\lambda+2\mu)]A_i=-\frac{P}{\pi k},$$

$$\sum_{i=1}^{3}[-\alpha_i(1-l^2k^2)+q_i(1-lk^2\alpha_iq_i)+lk^2q_i(1-q_i^2)]A_i=0, \quad (15)$$

$$\sum_{i=1}^{3}q_i(\alpha_i+q_i)A_i=0.$$

It is known that the effect of couple stresses especially affects at short waves [18]. Therefore, we introduce a dimensionless small parameter $\varepsilon=1/lk$.

With accuracy up to the order of magnitude ε^2 we obtain from:

$$q_1 = a_{01}\varepsilon, \quad q_2 = \frac{a_{02} + a_{12}\varepsilon}{\sqrt{\varepsilon}}, \quad q_3 = \frac{a_{02} - a_{12}\varepsilon}{\sqrt{\varepsilon}}, \tag{16}$$

where

$$a_{01} = -\frac{\gamma\sqrt{1-\beta_0^2}}{\sqrt{2}}, \quad a_{02} = -\frac{1+i}{\sqrt[4]{128}}, \quad a_{12} = \frac{i(1+i)(3-\beta_0^2)\sqrt{2}}{2\sqrt[4]{8}}, \quad \beta_0^2 = \frac{\rho D^2}{\lambda+2\mu}. \tag{17}$$

From (16), (14) and (15) we obtain the following relations:

$$\begin{gathered} \alpha_1 = \alpha_{01} + \alpha_{11}\varepsilon, \quad \alpha_2 = q_2, \quad \alpha_3 = q_3, \quad A_1 = -\frac{P}{\pi\lambda k}, \quad A_2 = A_3 = 0, \\ \alpha_{01} = \frac{(1+\beta_0^2)\gamma^2 - 2\beta_1^2}{\gamma\sqrt{1-\beta_0^2}}, \quad \alpha_{11} = \gamma\sqrt{1-\beta_0^2}\left[\frac{(1+\beta_0^2)\gamma^2}{2} - \beta_1^2\right], \end{gathered} \tag{18}$$

allowing to write the displacements (13) in the form:

$$\begin{gathered} u = -\frac{P}{\pi\lambda} \mathrm{e}^{-\beta_2 y} \int_{k_0}^{\infty} \frac{\sin kx}{k} dk, \\ v = -\frac{P\alpha_{01}}{\pi\lambda} \mathrm{e}^{-\beta_2 y} \int_{k_0}^{\infty} \frac{\cos kx}{k} dk - \frac{P\alpha_{11}k_0}{\pi\lambda} \mathrm{e}^{-\beta_2 y} \int_{k_0}^{\infty} \frac{\cos kx}{k^2} dk, \end{gathered} \tag{19}$$

where $k_0 = 1/l$, $\beta_2 = a_{01}k_0$.

Formulae (19) show that the displacement amplitudes will vary depending on the speed of a source D and magnitude of the load P created by it.

Figure 1 presents the dependence of normalized amplitude of transverse displacement

$$V_1 = -\frac{v\pi\lambda e^{\beta_2 y}}{P} = \alpha_{01} \int_{k_0}^{\infty} \frac{\cos kx}{k} dk + \alpha_{11}k_0 \int_{k_0}^{\infty} \frac{\cos kx}{k^2} dk$$

on the dimensionless speed squared of a moving disturbance source. The graphic image shows that as the velocity of a moving source approaches the velocity of a shear wave speed V_1 it increases without limit.

Figure 2 presents the dependences of amplitudes of transverse $v/v|_{y=0}$ (curve *1*) and longitudinal $u/v|_{y=0}$ (curve *2*) displacements on the depth.

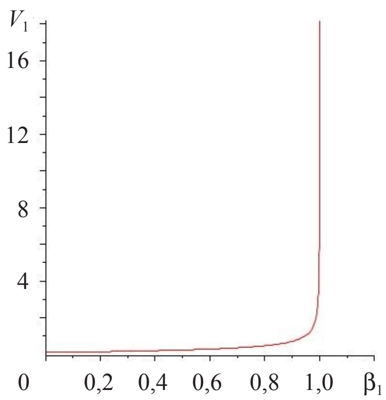

Fig. 1

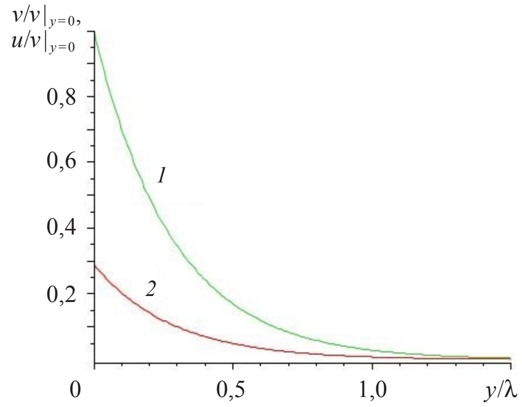

Fig. 2

The curves are given in dimensionless form: the displacement amplitudes are related to the normal displacement amplitude on the surface. The depth is plotted in fractions of wavelength. It is seen from the graphic image that the displacement components decrease exponentially with depth, the transverse component of the displacement vector always exceeds the longitudinal one, and the particles rotate as the surface wave propagates in the direction of the x axis along an elliptical path.

Conclusion

Based on the studies performed, it can be concluded that the source moving at a constant subsonic speed along the border of a gradient-elastic half-space will generate surface elastic waves. Such waves, in contrast to classical Rayleigh surface waves, have a dispersion. The displacement amplitudes change depending on the load of the moving source, as well as on its speed, and increase without limit as the source velocity approaches the shear wave speed.

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ГЕНЕРАЦИЯ ВОЗМУЩЕНИЙ СОСРЕДОТОЧЕННЫМ ИСТОЧНИКОМ, ДВИЖУЩИМСЯ С ПОСТОЯННОЙ ДОЗВУКОВОЙ СКОРОСТЬЮ ВДОЛЬ ГРАНИЦЫ ГРАДИЕНТНО-УПРУГОГО ПОЛУПРОСТРАНСТВА*

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В рамках математической модели градиентно-упругого континуума, то есть среды, напряженно-деформированное состояние которой описывается тензором деформаций, вторыми градиентами вектора перемещений, несимметричным тензором напряжений и тензором моментных напряжений, рассматривается задача о генерации возмущений движущимся источником. Изучаемая модель принадлежит к классу обобщенных континуумов, ее появление связано с именами Ж.-М. Леру и Т. Джеремило. К модели градиентно-упругой среды сводится и знаменитая модель континуума Коссера, когда в ней жестко зафиксирована зависимость вектора поворота от ротора перемещения (стесненное вращение). Предполагается, что источник движется с постоянной скоростью вдоль границы полупространства. Задача рассматривается в двумерной постановке, когда все процессы однородны вдоль горизонтальной поперечной координатной оси. Вектор перемещений содержит две компоненты: продольную и вертикальную поперечную. Скорость источника не превосходит по своей величине скорости сдвиговой упругой волны (дозвуковое движение). В результате аналитических исследований показано, что движущийся источник будет генерировать волны, распространяющиеся вдоль границы полупространства и экспоненциально убывающие в его глубину. Поперечная составляющая вектора перемещений всегда превосходит продольную, а вращение частиц при распространении возмущения происходит по эллиптической траектории. Такая волна, в отличие от классической поверхностной волны Рэлея, обладает дисперсией, поскольку ее фазовая скорость не является постоянной величиной, а зависит от частоты. Амплитуды перемещений изменяются в зависимости от величины нагрузки движущегося источника и его скорости. При приближении скорости источника к скорости сдвиговой волны амплитуды возмущений неограниченно возрастают.

Ключевые слова: градиентно-упругое полупространство, движущийся источник, поверхностная волна.

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