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CHARACTERIZING PANTOGRAPHIC SHEETS BY MEANS OF COMPUTATIONAL EXPERIMENTS

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Metamaterials is the new class of materials which are able to show a desired behavior at a macroscopic level. In the last years, properties of pantographic fabrics have been studied in the metamaterial framework. These systems are made of two families of orthogonal fibers, jointed in the intersection points by pivots. Such systems show interesting mechanical properties. The fibers and the pivots react differently to strain. In particular, bending and extension deformations involve only fibers (which are straight in the reference configuration), whereas torsion involves pivots. Such metamaterials is able to undergo large macroscopic deformation with small deformation of its microstructure. Since it is able to store a large amount of energy also beyond the elastic regime and it undergoes plastic deformation before rupture, it shows high toughness. Furthermore, an advantageous strength to weight ratio is observed. To predict the macroscopic behavior of these materials an efficient computational discrete model is needed. In this work, we will present a new numerical study of 2D pantographic fabric, in which its macroscopic behavior is modeled in terms of a second gradient continuum theory obtained by means of a heuristic homogenization procedure. In particular, we will consider a bi-axial extension test. We will discuss the distribution of the internal stored energy and we will compare the results with the uni-axial case.

Keywords: pantographic fabrics, deformations, bending, extension, torsion, heuristic homogenization, gradient continuum theory, computational experiments.

Introduction

The recent development of rapid prototype techniques like 3D printing has enhanced the design possibilities of new materials able to show a desired behavior at macroscopic level: the so-called metamaterials [1, 2]). A remarkable example is the pantographic sheet proposed in [3] which consists of two families of orthogonal fibers, jointed in the intersection points by pivots. This system shows interesting mechanical properties. Indeed, this metamaterial

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is able to undergo large macroscopic deformation with small deformation of its microstructure. Since it is able to store a large amount of energy also beyond the elastic regime and it undergoes plastic deformation before rupture, it shows high toughness. Furthermore, an advantageous strength to weight ratio is observed [3, 4], Although the idea of such a microstructure is well-known (see examples of pantographic mechanism in fibers or fabrics in [5–14] with the considered pantographic sheet it is possible to optimize the geometrical and morphological properties of the microstructure to obtain the desired overall mechanical behavior. A natural way to model the micro-structure of a pantographic sheet is to consider fibers as beams and to introduce a simple model for the pivots able to take into account their role during macroscopic shear deformations. Numerical results of this kind of models can be found in [15–17] On the other hand, in parallel to the design of the micro-structure, a continuous theory able to forecast the macroscopic behavior of the pantographic sheet is required. From the mathematical point of view, to find the continuum homogenized limit model associated to a certain microscopic discrete model is a complicated task that requires the employment of powerful mathematical tools (see for instance [18–20]). In the case of pantographic sheet, macroscopic behavior is modeled in terms of the second gradient theory [7, 21–24] obtained by heuristic homogenization (for more details see also [25]).
However, a computational efficient discrete model based on the micro-structure can be developed to forecast the macroscopic behavior of the pantographic sheet. Indeed, the discrete Hencky-type model proposed in [15, 16, 26] yields compatible results with the aforementioned continuum model but with an enhanced computational efficiency. One of the most interesting perspectives of this kind of analysis is the study of rupture mechanism in complex metamaterials (see also [27]). Currently, the research on pantographic metamaterials has several interesting open problems. It is worth to mention the study of the limit case of the inextensible orthogonal fibers, characterized by the possibility to decompose the placement field in two vector fields each depending only on the arch length along one family of fibers [28–32]. Moreover, the analysis of imperfections in the micro-structure, modeled as geometric randomly distributed defects, is of great practical interest. Finally, the study of dynamical aspects involving contribution from higher-gradient theory is still an open problem. In particular, it is of interest the observed difference with respect to first gradient orthotropic laminae concerning the natural eigenfrequencies [33].

Formulation of the problem

Let us consider a pantographic sheet, i.e. a planar network of two families of orthogonal fibers. The fibers are equally spaced and connected in the intersection points by elastic pivots around a perpendicular axis with respect to the network plane. In order to introduce the second gradient model, let us consider a continuous homogenized 2D elastic surface obtained by the aforementioned micro-structure. This surface is characterized by a potential energy U which depends on the deformation gradient tensor, $\mathbf{F}$, and its gradient $\nabla\mathbf{F}$, both evaluated along the fiber directions $\mathbf{D}_1$ and $\mathbf{D}_2$ [25]:

$$U = \int_\Omega \sum_\alpha \frac{K_e^\alpha}{2}(\|\mathbf{F}\mathbf{D}_\alpha\| - 1)^2 \, d\Omega +$$

$$+ \int_\Omega \sum_\alpha \frac{K_{eb}^\alpha}{2}\left[\frac{\nabla\mathbf{F}|\mathbf{D}_\alpha \otimes \mathbf{D}_\alpha \cdot \nabla\mathbf{F}|\mathbf{D}_\alpha \otimes \mathbf{D}_\alpha}{\|\mathbf{F}\mathbf{D}_\alpha\|^2} - \left(\frac{\mathbf{F}\mathbf{D}_\alpha}{\|\mathbf{F}\mathbf{D}_\alpha\|} \cdot \frac{\nabla\mathbf{F}|\mathbf{D}_\alpha \otimes \mathbf{D}_\alpha}{\|\mathbf{F}\mathbf{D}_\alpha\|}\right)^2\right] d\Omega +$$

$$+\int_{\Omega}\sum_{\alpha}\frac{K_p}{2}\left[\sin^{-1}\left(\frac{\mathbf{FD}_\alpha}{\|\mathbf{FD}_\alpha\|}\cdot\frac{\mathbf{FD}_\alpha}{\|\mathbf{FD}_\alpha\|}\right)\right]^2 d\Omega$$

with α taking values 1 and 2 for the two families of fibers and K_e^α, K_b^α and K_p being material stiffnesses related to stretching, bending and shear deformation, respectively. Let us remark that the first addend involves fibers' elongation along their own directions; in the second addend the only contribution of the second gradient is to the Lagrangian curvatures, namely the rates of change of the current tangent vectors to the fibers with respect to arc length along the same fiber in the reference configuration; finally, in the third addend the angle between the two different families of fibers appears.

Numerical simulation

The behavior of the system under a bi-axial elongation test is studied by means of Finite Elements Method. The equilibrium shapes are obtained by seeking the configuration of the system that minimize the aforementioned energy U and which satisfies the associated boundary conditions. The particular system under analysis is a square with a 178 cm long edge; each vertex is chamfered in order to have a shortest edge of 42 mm (see the black solid lines in fig. 1). In the reference configurations, the two fibers are disposed along x- and y-axes. Material constant employed are $K_e = 1.34\times10^5$ Nm^{-1}, $K_b = 1.92\times10^{-2}$ Nm and $K_p = 1.59\times10^2$ Nm^{-1}. The system undergoes a uniform increasing displacement to the shortest chamfered edges along the orthogonal direction with respect to them. The resulting equilibrium shape, in the case of a displacement of 42 mm, is plotted in fig. 1.

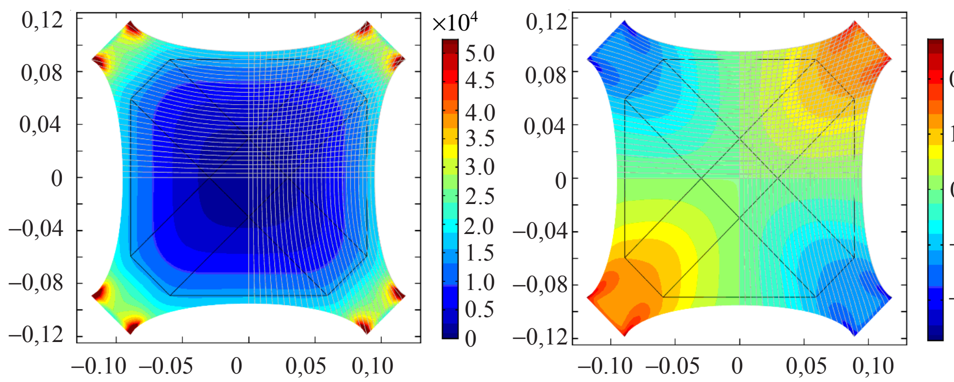

Fig. 1. Biaxial test: Equilibrium shape. The colors indicate the energy density on the left panel, and the shear deformation on the right panel. Black solid lines indicate the reference configuration. Grey solid lines indicate some material lines in the present configuration

In such a configuration, the strain energy density is plotted in the colored map in the left panel whereas the shear deformation is given in the right panel. Grey lines indicate local orientation of fibers.

The resultant reaction magnitude versus the imposed displacement on one of the loaded edges are displayed in fig. 2.

Figure 3, instead, shows the distribution of the reaction components along the loaded end in the orthogonal (R_1) and parallel direction (R_2).

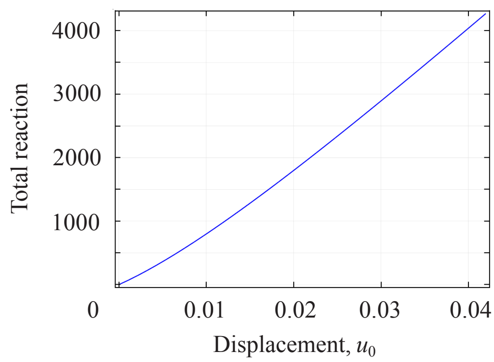

Fig. 2. Total reaction on one of the shortest edges

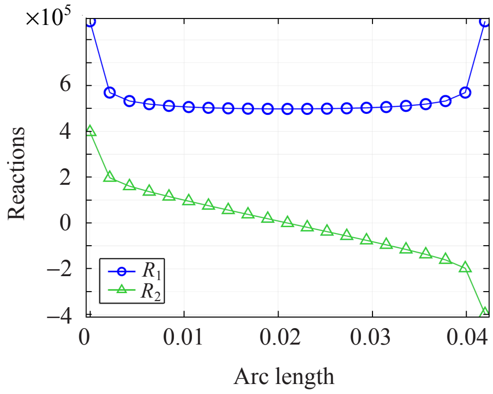

Fig. 3. Distributions of the reaction on one of the shortest edges along the normal (R_1) and tangent direction (R_2)

The total reaction is mostly linear for a huge deformation range. Furthermore, stress is clearly localized at the edges end points, namely where the displacement is imposed, as it is observed in the uni-axial case.

Conclusions

In this work, we have analyzed the specific mechanical behavior of a particular class of metamaterials called pantographic sheets. This metamaterial exhibits a micro-structure characterized by both a high contrast ratio between bending and extensional stiffness, while its macroscopic behavior shows a high contrasted gradient of displacement in the axial and transverse direction. Another source of anisotropy is given by the presence of two preferred material directions of a high extensional stiffness that give rise to an orthotropic material. This complex microstructure produces the onset of internal boundary layers where gradients of deformation come out. It is therefore useful to adopt generalized continuum theory in order to have a satisfactory predictive power without unpractical computational costs (as the ones required by employing first gradient Cauchy models).
New numerical simulations on a 2D second gradient non-linear continuum model allowing for planar motion only have been shown and discussed. By means of Finite Elements Method, in particular, we have analyzed the behavior of a pantographic sheet undergoing a bi-axial extension deformation. Like in the uni-axial case, the internal stored energy is

mostly concentrated in the clamping edges corners with a uniform distribution around the sample center. The main difference with the uni-axial case is observed in the distribution of shear deformation energy that vanishes in the central region due to the symmetry of the test.

Since pantographic sheets can describe plates and shell characterized by additional degree of freedom (as discussed for instance in [34–38]) it is possible to consider applications of this analysis to this framework. Indeed, like shell or plate, pantographic sheets are flexible and thin while the main difference is in its flexibility in case of shear deformations.

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ОПИСАНИЕ ПАНТОГРАФИЧЕСКИХ ЛИСТОВ С ПОМОЩЬЮ ЧИСЛЕННЫХ ЭКСПЕРИМЕНТОВ

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Метаматериалы являются новым видом материалов, демонстрирующих желаемое поведение на макроскопическом уровне. В последние годы свойства пантографических структур изучались с точки зрения метаматериалов. Подобные системы состоят из двух семейств ортогональных волокон, соединенных в точках пересечения с помощью шарниров. Эти системы обладают интересными механическими свойствами. Волокна и шарниры по-разному реагируют на деформации. В частности, деформации изгиба и растяжения затрагивают только волокна (так как они являются прямыми в референсной конфигурации), тогда как кручение затрагивает шарниры. Подобные метаматериалы способны претерпевать большие макроскопические деформации с малыми деформациями своей микроструктуры. Поскольку такие материалы способны накапливать большое количество энергии также за пределами упругого деформирования и подвергаются пластической деформации перед разрывом, они демонстрируют высокую ударную вязкость. Кроме того, следует отметить их высокую удельную прочность. Для того чтобы прогнозировать макроскопическое поведение подобных материалов, требуется эффективная численная дискретная модель. Представлен новый численный анализ двухмерной пантографической структуры, макроскопическое поведение которой моделируется в терминах теории континуума со вторыми градиентами, полученной с помощью эвристической процедуры усреднения. В частности, рассмотрен пример двухосного растяжения. Обсуждается распределение внутренней накопленной энергии и проводится сравнение результатов с одноосным случаем.

Ключевые слова: пантографические ткани, деформации, изгиб, растяжение, кручение, градиентная континуальная теория, вычислительные эксперименты.